

## On Teaching Primary Calculus: First Step in a Design Research and some Issues

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Change is everywhere; all day long we encounter change. Understanding change is important to understand and manipulate the changing world around us. Traditionally change is taught formally as calculus in secondary education and up to students that already do know functions and algebra. Furthermore, in traditional calculus an instantaneous rate of change is defined as the limit of the average rate of change on an interval, as that interval approaches length zero. Advanced mathematical concepts like limit and algebra are unsuitable for primary education. However, ICT enables primary students to reason about change without the need to know these mathematical concepts. Using ICT and an alternative to the standard mathematical definition of instantaneous rate of change, change can already be taught formally in primary school. To denote teaching calculus-like concepts to primary students we introduce the term “primary calculus”. The aim of this research is to develop an understanding on how to teach primary calculus to 5th grade.

To that end a design research (Gravemeijer & Cobb, 2006) is carried out to develop an inquiry-based and ICT-rich instructional sequence on primary calculus for 5th grade that can be integrated into the curriculum and is close to the world-view of 5th graders and a local instruction theory on that instructional sequence. In the preparatory phase of a design research three steps are taken: (1) defining the learning goals, (2) determining the instructional starting points, and (3) conjecture a first local instruction theory on how to get from the starting points to the learning goals. All three steps are based on results from relevant literature.

Teaching calculus early is not a new idea. Since computers entered education at a larger scale in the 1980s, there have been a number of research projects on teaching calculus-like concepts to primary or middle school students using computers. Common among these projects is the use of graphs to represent change as graphs are the main communication method of calculus (Boyd & Rubin, 1996; van Galen & Gravemeijer, 2010; Doorman & Gravemeijer, 2009). Although using graphs is not straightforward (Leinhardt, Zaslavsky, & Stein, 1990; Ainley, Pratt, & Nardi, 2001; Lowrie & Diezmann, 2007), supported by ICT young students are quite capable using graphs (Phillips, 1997), especially given their lack of graphing experience. Other common characteristics that are more problematic are the preference for teaching average rate of change first and the use of the motion domain as problem domain. Using different problem domains and teaching a more intuitive notion of instantaneous rate of change without a dependency on average rate of change is advantageous for learning primary calculus.

Not much is known about what primary students already do know about change. To determine the instructional starting points a preliminary study was performed. Carlson, Jacobs, Coe, Larsen, and Hsu (2002) developed the covariation framework to analyze college students’ reasoning about two co-varying quantities in a situation of change. By using a suitable microworld, this framework can be adapted for analyzing 5th graders’ reasoning about change. Based on this adapted framework an one-on-one teaching experiment (Steffe & Thompson, 2000) and suitable microworld were developed. In 8 experiments 9 participants had to solve increasingly more difficult problems about filling glasses with water. Through instant feedback of the microworld, the participants seem able to break easily through the linearity illusion (de Bock, van Dooren, Janssens, & Verschaffel, 2002) and express their understanding of filling of an Erlenmeyer flask.

Over all tasks and all problems the participants exhibited a level of reasoning about change indicating their understanding of the direction of change and the amount of change at certain points and intervals. The participants did not reason about change in terms of rate of change.

But is this a reasonable indication of 5th graders' current level of reasoning about change? Can we determine 5th graders' previous knowledge about change with this preliminary study or is it a reflection of what they are able to learn during this experiment? And what is the value of ecological validity when using an innovative microworld? It is unlikely practitioners will have experience with something similar.

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