Research Proposal

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Contents

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1.2 Title of research proposal (NL and UK)

NL: Hoe kunnen we leerlingen in groep 7 het begrip 'samengestelde variabele grootheid' onderwijzen in verschillende domeinen?
UK: How can we teach 5th graders the concept of a compound variable quantity in different domains?

1.3 Abstract

Preparing pupils for successful participation in our highly technological information society requires the introduction of new science and technology education. This includes the mathematics of change and variation (MCV) of which the concept of compound variable quantities is part. Traditionally, MCV is taught in secondary education as calculus by using algebra. Using dynamic representations makes it possible to start teaching MCV in primary education.

The aim of this project is to study the teaching of the concept of compound variable quantities in primary education. Specifically, we ask the question how we can teach 5th grade pupils the concept of a compound variable quantity in different domains of science and technology. We apply a design research-approach by means of which, based on theory, an instructional sequence to teach 5th graders the concept of a compound variable quantity is developed. We hypothesize that by using dynamic representations pupils are able to learn about compound variable quantities in different domains.

1.4 Keywords

Calculus, Design Research, Primary Education, Science and Technology Education

2 Research proposal

2.1 Research topic

Over the past decades our society has changed drastically. Science and technology, in particular information technology, play an important role in our daily lives. To be able to participate in this highly technological society, recognition and understanding of the underlying scientific world view is key. To prepare our pupils for their role in future society suitable science and technology education is essential (Gravemeijer, 2009; van Keulen, 2009).

The reality in our schools does not reflect this need: science and technology education in primary schools, particularly in the Netherlands, is often limited to a small number of hands-on activities once a month (van Keulen, 2009). According to the Inspectorate of Education of the Netherlands in 2009, only 19% of Dutch primary schools did achieve a satisfactory level of science and technology education (Inspectie van het Onderwijs, 2010, p. 45). Although this is an improvement over the percentage of primary schools performing satisfactory on science and technology in 2004 (2%) (Inspectie van het Onderwijs, 2005, p. 27), science and technology education in Dutch primary school is still highly underdeveloped.

Teachers often have a limited understanding of science and technology. They do not feel safe teaching science and technology and resort to what they know from their own experiences. Consequently classes labeled "science and technology" are often workshop classes (Léna, 2006; van Keulen, 2009) where pupils execute traditional science experiments or tinker with well-known materials and tools such as wood, nails, and hammer, cardboard, glue, and scissors.

In this context many a researcher voiced the need for new science and technology education in primary education (Gravemeijer, 2009; van Keulen, 2009; Millar & Osborne, 1998; Léna, 2006). This new science and technology education is based on the observation that pupils are curious about the world around them. Science and technology education can build upon that curiosity by giving pupils new means to explore their environment: "It endows them with a rich understanding of our complex world, helps them practice an intelligent approach to dealing with the environment and develops their creativity and critical mind, their understanding of reality, compared to virtuality and teaches them the techniques and tools that societies have used to improve the human condition." (Léna, 2006, p. 8)

To accomplish this, the following characteristics have been brought to the fore for innovative science and technology education. First off, science and technology education should be inquiry-based. Inquiry-based education makes pupils learn by involving them in real-life situations close to their point of view with an emphasis on questioning, hypothesizing, and experimenting by the pupils themselves (Léna, 2006; Rocard et al., 2007; Osborne & Dillon, 2008).

At the same time science and technology education in school should be close to the world view of the pupils. Their environment has almost no relation to the subculture of science. New science and technology education should try to minimize this gap between pupils' realities and the reality propagated by the scientific world view (Osborne & Dillon, 2008).

Furthermore, new science and technology education should use modern information and communication technology (ICT). Not only will pupils grow up in a society where ICT is ubiquitous, the use of ICT also has an enormous potential in education itself. It enables new ways of teaching and learning. (Osborne & Hennessy, 2003; Murphy, 2003; Bingimlas, 2009; Woodgate, Fraser, & Crellin, 2007)

Finally, science and technology education should be integrated into other subjects in the curriculum. The curriculum is already overflowing so it is difficult to fit in a separate subject science and technology. And as science and technology are an integral part of our society so should it also be an integral part throughout the curriculum (van Keulen, 2009).

In this research we center on the question to better understand how to bring these characteristics of innovative science education in practice. Of course, as science and technology education is an extensive domain, we can only focus on a sub domain of science and technology education. We limit ourselves to the education of the mathematics of change and variation (MCV) in science and technology contexts.

MCV is an important part of the scientific world-view and a good understanding of MCV prepares pupils better for participation in our highly scientific and technological society. The education of MCV is also more challenging than most of the current science and technology education in Dutch primary schools. Fortunately ICT can be put to good use to teach science and technology. ICT is extremely suited to measure, record, and visualise change (Tinker, 1999), MCV is cut out for the use of ICT in education. In the next section the concept of mathematics of change and variation is elaborated in more detail.

2.1.1 The mathematics of change and variation

The world around us changes continuously. Change is everywhere, from the ever moving planets in our solar system to simple daily activities, such as when heating water to brew tea. Although most often implicit and unnoticed, change and variation are an important part of our lives. We often unconsciously act in reaction to change – while brewing tea, we know to wait turning off the heat until the water starts to boil –, in other situations understanding change helps us to make more significant decisions, like selecting a mobile plan or getting a mortgage. From childhood on, literally from the first steps we take, change is part of our lives.

Not surprisingly, "change" is an millennia old philosophical topic. During the scientific revolution in the scientific world-view the concept of change became to mean a relation of a property of an object in consecutive points in time (Mortensen, 2008). This relation can be described mathematically by a continuous function (of time).

As change is ubiquitous in our daily lives, pupils do have a mostly intuitive understanding of the concept of change. They will try to understand the world around them using this early understanding. However, this intuitive concept differs from the formal scientific concept of change (Doorman, 2005; McCloskey, 1983). Overcoming these differences challenges teachers and pupils alike (McCloskey, 1983; Parnafes, 2007).

Change becomes visible through comparisons of one point in time with an earlier (or later) point in time and observing differences between the two points in time: certain qualities (of an object) change over time. We call these situations *dynamic systems*. From a scientific perspective we can understand change in dynamic systems by analysing a mathematical model of these systems, Qualities (and not the objects themselves) are quantified through measuring: by assigning a numerical value to a quality, qualities become quantities (Thompson, 1994b). According to Jones these quantifiable qualities correspond to variables (Jones, 1971). A mathematical model describes the changing quantities – we call them varying quantities – using formulae denoting relationships between the dependent quantities.

Often we encounter situations wherein the change of one quantity depends on change of another quantity. Take for example the dynamic system of taking a bath:

> A bathtub is filled for three quarters with (too) hot water. Over time the temperature of the water will decrease. How long do I have to wait until the water has reached a bearable temperature? And once I step into the bath tub for how long can I enjoy a pleasant warm bath?

In this example the dynamic system consists of a partially filled bathtub with warm water. As the air and the tub conduct heat, the warmth of the bathwater slowly flows away: the water cools down.

The corresponding mathematical model of the dynamic system "taking a bath" would look something like $T = f(t)$, with temperature of the water in the tub in $^{\circ}C$ T and time passed in minutes t. [1] The function f denotes that the value of T depends on the value of t. The exact formulation of this function is irrelevant for now.

A mathematical model of a dynamic system can be a linear function or a system of equations involving two co-varying quantities. Although change is perceived through comparison of quantities at different points in time, it is possible to model a dynamic system without the quantity time. The cooling down of the bath water in the example can be accelerated by adding cold water. We can model this dynamic system by relating the change of temperature of the bath water with the volume of cold water added. Now volume is the independent variable instead of time.

To understand change in this situation more thoroughly, we can draw the graph of f as a continuous curve over a certain domain (see Figure 1, page 6). The graph denotes the course of change in this situation. As the graph is a continuous curve, the speed of temperature change varies continuously: at every point on the graph there is an instantaneous speed of temperature change. We call such an instantaneous speed a (new) *compound variable quantity*. Compound variable quantities are also known as a rate.

The term "rate" is closely linked to the term "ratio" as these terms are used as synonyms in common day use. The two terms do have a different meaning, however. Patrick Thompson defines "ratio" as 'the result of comparing two quantities multiplicatively' (Thompson, 1994b, p. 17) and a rate as 'a reflectively abstracted constant ratio' (Thompson, 1994b, p. 18). Thompson here refers to Piaget's notion of reflexive abstraction. In this process of reflexive abstraction of a ratio one first learns that a rate exists and then one understands it to be a (new) quality of an object that can be measured and quantified as any other variable quantity. Compound variable quantity then becomes a mathematical object one can reason with. Constructing a compound variable quantity as a mathematical object is a difficult process (Thompson, 1994b, 1994a).

In a graph we determine the instantaneous speed at a point as the tangent of the graph at that point. In a formula we denote the instantaneous speed as the derivative of f. Functions and calculus are advanced mathematical concepts part of the domain of the mathematics of change and variation (MCV).

[1] Taking a bath is modelled using Newtons Law of Cooling: $T(t) = T_{room} + (T(0) - T_{room})e^{-kt}$. The initial temperature $T(0)$ is $60^{\circ}C$ and the room temperature T_{room} is $20^{\circ}C$. To calculate k it is assumed that after 45 minutes the water in the bath has cooled down to $25^{\circ}C$. This is a simplified model.

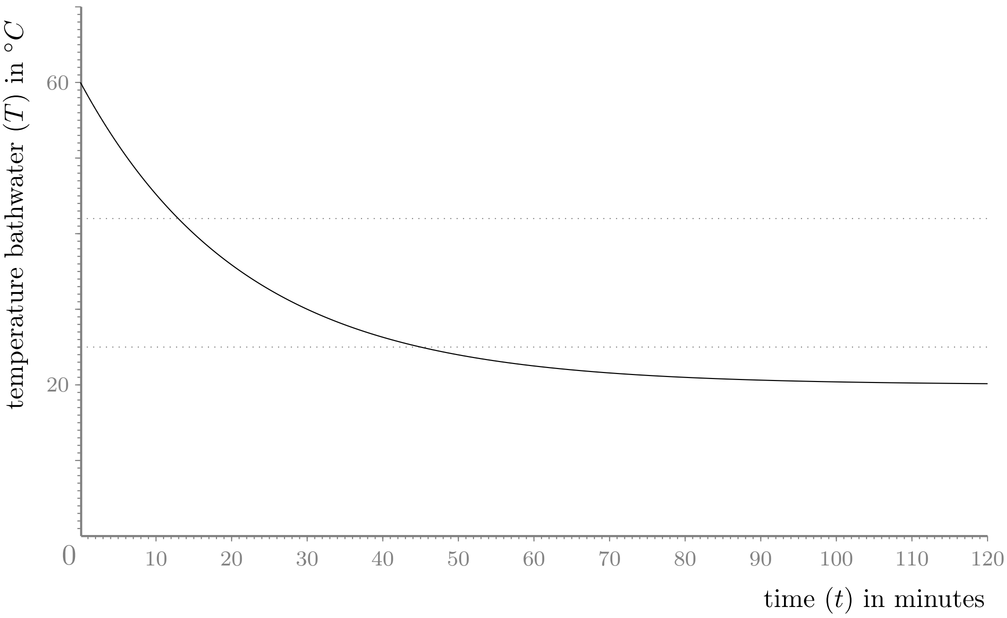

Figure 1: Dynamic system "taking a bath" represented in a graph. The area between the two horizontal dotted lines represents the "zone of comfortable water temperature to take a bath". Above $42°C$ the water is too hot and below $25°C$ the water is not warm enough to enjoy the bath any more.

2.1.2 The mathematics of change and variation in primary education

Pupils' understanding of the concept of change is studied most in the domain of motion. Through experience pupils have build a coherent theory supporting most encounters with motion in their daily lives. According to McCloskey these intuitive theories often are consistent with pre-Newtonian theories (McCloskey, 1983). As such these theories 'are incompatible with established scientific theory [and] are quickly labeled as "misconceptions" and dismissed by most scientists. (...) Indeed, every one of the misconceptions about motion common among students today was seriously advocated by leading intellectuals in pre-Newtonian times. (...) If the evaluation of common sense was so difficult for the intellectual giants from Aristotle to Galileo, we should not be surprised to find that it is a problem for ordinary students today.' (Halloun & Hestenes, 1985, p. 1065)

Current scientific explanations of motion are closely linked to calculus. Traditionally MCV is first taught as calculus in secondary education and beyond. Besides overcoming these problematic intuitive theories of motion, learning calculus adds its own set of problems to understanding change.

Carlson et al. (Carlson, Oehrtman, & Engelke, 2010) however report that many pre calculus students do have an action view of function: they are able to manipulate the symbolic formula or interpret the graph geometrically, but are unable to understand the dynamic situation underlying these representations. A better understanding of compound variable quantities would give students a better foundation to learn the concept of function and calculus related concepts (Carlson et al., 2010; Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Thompson, 1993). Almost all pupils in secondary education in the Netherlands are taught simple calculus related concepts. To give pupils a good foundation to learn these concepts in secondary education, the concept of compound variable quantities could already be explored in primary education.

According to (Stroup, 2002) teaching mathematics of change – he calls it "qualitative calculus" in contrast to the quantitative nature of traditional calculus – is a worthwhile undertaking because qualitative calculus is not only a transitional phase learning traditional calculus but valuable knowledge in itself. Stroup calls compound variable quantities "'how fast' quantities" referring to a speed of change and the related depended variable quantity a "'how much' quantity" denoting the total amount of change. He coined the term *qualitative calculus* to be "the intensification of rate and two kinds of reversibility between what are called "how much" (amount) and "how fast" (rate) quantities are what interactively and collectively characterize and to help to define understanding qualitative calculus."(Stroup, 2002, p. 168)

Early knowledge of MCV not only eases the learning and understanding of mathematical concepts like function and calculus, it is also beneficial for all as MCV contains ‘the very concepts children most need not only to participate in the physical, social, and life sciences of the twenty-first century, but also to make informed decisions in their personal and political lives.’(Roschelle, Kaput, & Stroup, 2000, p. 2) Kaput and Roschelle state that access to MCV historically is limited to10% of the (American) population. They set out to democratise access to MCV using ICT (Kaput & Rochelle, 1998; Roschelle et al., 2000; Kaput & Schorr, 2007).

In the 1980s MCV entered the domain of primary education and early secondary education through the capabilities of then modern computing equipment. Using a computer students can experiment with dynamic representations of models of dynamic systems. Since the late 1980s many a researcher studied computer aided education of MCV (Parnafes, 2007; Doorman & Gravemeijer, 2009; van Galen & Gravemeijer, 2010; Boyd & Rubin, 1996; Kaput & Schorr, 2007; Stroup, 2002; Herbert & Pierce, 2008; Wilhelm & Confrey, 2003).

The basic premise is that ICT enables new representations of mathematical concepts overcoming the need to learn algebra first to start learning calculus. These new representations are dynamic in nature and use extensively graphical representations (Kaput & Rochelle, 1998) as the graph is the main communication method of calculus (Boyd & Rubin, 1996; van Galen & Gravemeijer, 2010; Doorman & Gravemeijer, 2009). To learn calculus graphs are indispensable. Students do have difficulties interpreting graphs as they see them as images, not as a representation of an underlying model (Roth, Bowen, & McGinn, 1999) Although more difficulties interpreting graphs are reported (Barton, 1997; Lowrie & Diezmann, 2007), there are also indications that young pupils are sufficiently able to read graphs (Phillips, 1997). Key in overcoming potential problems with interpreting graphs by pupils is how graphical representations are introduced: the context of the situation graphed and the use of ICT play an important role (Noble, Nemirovsky, Dimattia, & Wright, 2004; Phillips, 1997; van den Berg, Schweickert, & Manneveld, 2009).

Dynamic representations enable learners to experiment with calculus related concepts without the need for an advanced mathematical foundation. As a result dynamic representations are suitable for learning calculus related concepts in situations like primary education where learners do not have a mathematical background.

The education of MCV in secondary education and up is a well-researched topic, especially in the domain of motion and speed in physics and mathematics. Actually, almost all research is related to these two subjects. How can research on motion and speed in education be used to educate mathematics of change in other domains?

Herbert and Pierce (Herbert & Pierce, 2008) studied the use of computer software for teaching about motion and speed. They also studied if the concepts learned transfered to a non-motion domain. Students were able to see similarities between different domains. (Wilhelm & Confrey, 2003) found that transfer of concepts about MCV between motion and non-motion domains was possible although not without problems as some students were unable to transfer understanding even with a deep understanding of the concept of change.

Most research on aspects of teaching and learning of compound variable quantities discussed so far are about students in secondary education and higher. About the teaching of compound variable quantities in primary school almost nothing is known, therefore the central research question is: *How can we teach 5th grade pupils the concept of a compound variable quantity in different domains?*

The main research problem will be divided into the following research questions:

1. What do 5th graders already understand about the concept of a compound variable quantity?
2. Given their initial understanding of the concept of a compound variable quantity what inquiry-based and ICT-rich instructional sequence that can be integrated in the curriculum and is close to their world-view can deepen 5th graders' understanding of compound variable quantities?
3. How can dynamic representations support the teaching of the concept of a compound variable quantity?
4. In what ways can teaching the concept of a compound variable quantity in different domains like motion, economics, biology, etc., improve the understanding of the concept of a variable compound quantity?

2.2 Approach

To answer these research questions design research is indicated. Design research is tailored to develop both theory and a new, innovative educational artifact of some sort (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collective, 2003; Shavelson, Phillips, Towne, & Feuer, 2003; Gravemeijer & Cobb, 2007). Design research fits our constructivist perspective on learning and acquiring knowledge. Furthermore it also fits the research problem as almost nothing is known about the teaching and learning of the concept of compound variable quantities in primary education. Design research enables us to develop both an innovative instructional sequence and a suitable theory on that innovative instructional sequence (Gravemeijer & Cobb, 2007).

Design research, as outlined by Gravemeijer (Gravemeijer & Cobb, 2006; Gravemeijer & Cobb, 2007; Gravemeijer, 2004), is an iterative process on two accounts. First, there are macro cycles consisting of three consecutive phases: preparation, teaching experiments, and a retrospective analysis. In a design research there can be more than one macro cycle. Second, the teaching experiments phase itself consists of micro cycles of designing part of an instructional sequence, the subsequent implementation and testing of that design in a real teaching situation, and a reflection on the micro cycle that forms the basis for the next micro cycle.

In the *preparatory phase* of a macro cycle first the educational end points of the instructional sequence to be developed are set. Then the focus is on the starting points: what do the students already know about this subject? Finally, based on the starting points and the educational end points an initial local instruction theory can be formulated. 'Such a conjectured local instruction theory consists of conjectures about a possible learning process, together with conjectures about possible means of supporting that learning process. The means of support encompass potentially productive instructional activities and (computer) tools as well as an envisioned classroom culture and the proactive role of the teacher.' (Gravemeijer & Cobb, 2006, p. 50) These conjectures are based on the existing literature and relevant research results from related projects like STEFF[2].

This initial local instruction theory forms the basis of the second *phase of teaching experiments* and is set up to allow it to be tested and refined in a number of micro cycles. Starting from the current local instruction theory part of an instructional sequence is designed and developed by conducting anticipatory thought experiments. Subsequently that instructional sequence is put into practice in a classroom teaching experiment. Afterwards, the design and experiment are evaluated by analysing what has happened in the classroom during the experiment. To that end, teachers' instructions, pupils' activities and classroom processes are monitored by video taping and observing the whole classroom teaching experiment.

The results of this analysis are used to improve the local instruction theory. This new local instruction theory will be the basis for the next micro cycle, and so on. Design research is an iterative process to refine both the local instruction theory and the instructional sequence.

Finally, in the last phase of a macro cycle, the *retrospective analysis phase*, the whole process of preparation and repeated experimentation is reflected upon to create an empirically grounded local instruction theory informing teachers and researchers alike to use in their classes or start further research.

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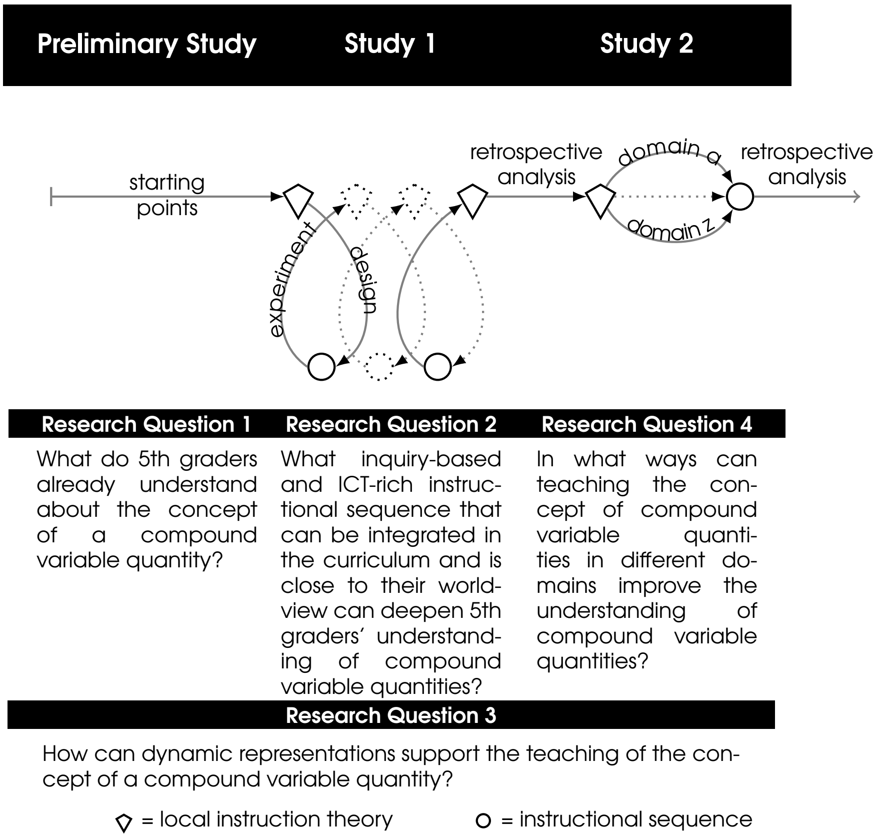
This cyclic design research approach propagated by Gravemeijer is applied to this research project (See Figure 2). However, before an initial local

[2]STEFF, Science and Technology Education For the Future, is project group consisting of members of ESoE, the Freudenthal Institute Utrecht and Avans University of Applied Sciences. The goal of STEFF is to create innovative science and technology education for primary education. My supervisors and I participate in STEFF.

Figure 2: The approach of the research using design research. The research is divided into three consecutive studies each focussing on one research question in particular.



instruction theory for the concept of compound variable quantities for 5th graders can be developed, the starting points should be clear. As there is no literature on what pupils in primary education do know about compound variable quantities, a preliminary study is first carried out to answer the first research question: What do 5th grade pupils already understand about the concept of a compound variable quantity? (See Figure 2, Preliminary Study). The approach of the preliminary study will be discussed in more detail in Section 2.2.1 (page 12).

Once 5th graders' previous knowledge on compound variable quantities is determined, the first macro cycle of the design research can be continued by the creation of an initial local instruction theory and subsequently with the second phase of repeated experimentation (see Figure 2, Study 1) consisting of a number of micro cycles to answer the second research question: given their initial understanding of the concept of a compound variable quantity what inquiry-based and ICT-rich instructional sequence that can be integrated in the curriculum and is close to their world-view can deepen 5th graders' understanding of compound variable quantities?

As result of the retrospective analysis of the first macro cycle an improved local instruction theory for the concept of compound variable quantities is formulated. That improved local instruction theory forms the basis for the next macro cycle of the design research focussing on the teaching of compound variable quantities in *different* domains (see Figure 2, Study 2) to answer the fourth research question: In what ways can teaching the concept of a compound variable quantity in different domains like motion, economics, biology, etc., improve the understanding of the concept of a variable compound quantity? The micro cycles in this study consist of developing, implementing, testing and evaluating instructional sequences to teach about compound variable quantities in different domains. At the end of the second study, the second macro cycle will be analysed and reflected upon, followed by a retrospective analysis of the whole process to complete the design research. As a result, the improved local instruction theory will be broadened in scope.

The general approach to studies 1 and 2 and its relation to the preliminary study is explicated in Section 2.2.2 (page 14).

2.2.1 Detailed approach of the preliminary study

Before we can start developing the first local instruction theory we have to know what pupils already do know about compound variable quantities. To capture 5th grade pupils' understanding of compound variable quantities a preliminary one-on-one teaching experiment is being developed (Steffe & Thompson, 2000). The goal of this teaching experiment is to determine at what developmental level 5th graders reason at in the covariation framework.

Carlson et al. developed a framework to analyse "covariational reasoning" (Carlson et al., 2002; Carlson et al., 2010). They define "covariational reasoning" as: 'the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other.'(Carlson et al., 2002, p. 354) Carlson et al. use their framework to analyse college students' understanding of dynamic systems with two dependent variable quantities.

The framework consists of five mental actions identified by analysing the behaviour of students solving problems about two co-varying variable quantities. The mental actions appear as the verbal and representational utterances of the students. As such, the mental action are discernible indirectly only. On top of these five mental actions, five developmental levels are distinguished. A student is on a certain level if he or she performs the corresponding mental action of that level and all mental actions of the lower levels. However, Carlson et al. warn for so called pseudo behavior (Vinner, 1997) as students can display behaviour of a certain level without being on that level of understanding.

To determine the mental actions of pupils, one-on-one teaching experiments are used. The idea of a one-on-one teaching experiment is to engage a pupil in some instructional activity and adapt the activity to capture best the pupils' mental actions. To that end the pupil is asked to think out loud and the pupil will be asked to explain their behaviour.

Pupils in primary education do not know algebra, functions, or calculus as do college students the framework is based on. Students use these mathematical concepts to build and interpret mathematical models of dynamic systems. A mathematical model can also be made accessible by a dynamic representation like a computer simulation of that dynamic system. According to (Kaput & Schorr, 2007) young pupils are able to understand quite complex dynamic systems using a dynamic representation as the pupils are able to explore, study, and analyse the underlying mathematical model. The covariation framework will be adapted for use in primary education through the application of dynamic representations.

In this first experiment 10 to 15 5th grade pupils will participate. In a one-on-one teaching experiment they are presented with some increasingly more difficult problems on compound variable quantities and are asked to solve them while thinking out loud. The researcher will continuously ask the pupils to explain their reasoning behind their behavior and thinking. The whole experiment will be videotaped to capture all utterances of the respondents, be they verbal, gestures, or representational.

After the experiments the videotapes will be transcribed fully and subsequently analysed using the covariation framework to determine the developmental level the respondents reason at on compound variable quantities. Furthermore insight in how to adapt and use the framework in studies 1 and 2 will be gained.

2.2.2 General approach of studies 1 and 2

Study 1 starts with the creation of an initial local instruction theory based on the outcome of the preliminary study. To answer the second research question – what is a inquiry-based, ICT-rich instructional sequence that is integrated into the curriculum and close to the world view of the pupils to deepen 5th graders' understanding of compound variable quantities? – this initial local instruction theory will be refined in the teaching experiment phase of the first macro cycle of the design research. This refinement is the result of an iterative process with a number of micro cycles of classroom experiments performed with one class (see Figure 2, study 1).

Every classroom teaching experiment is prepared with input and participation of the teacher doing the experiment in his or her classroom and is based on the current local instruction theory. The instructional sequence to be developed for each classroom teaching experiment is modular in nature. Each subsequent module contains only a small number of instructional activities that make up one lesson. The teacher is trained to teach the module.

During and after the execution of a module different kinds of data are gathered: every lesson will be video taped; artefacts created by pupils are collected; sometimes pretests, posttests, or tests are given and collected; sometimes pupils are interviewed; afterwards the lesson given will be evaluated by the teacher and researcher through a video or audio taped interview or meeting. Audio and video tapes will be transcribed. These data are analysed and are input for the next micro cycle: for the development of the next module and for the refinement of the current local instruction theory. Carlson et al.'s adapted covariation framework is used in the analysis.

The whole instructional sequence will contain around five modules; there will be around five micro cycles in study 1. The respondents in study 1 are one fifth grade teacher and his or her class. Given time and opportunity another teacher and class will be involved in this study starting another macro cycle of design research.

Study 1 finishes with a retrospective analysis of the design process up to that point incorporating the preliminary study and the micro cycles of the teaching experiment. This will result in a local instruction theory as input for study 2 and an improved version of the adapted covariation framework.

In study 2 instructional sequences to teach the concept of a compound variable quantity in at least three different domains are developed (see Figure 2, Study 2) consisting of a small number (one or two) modules. Each module is developed, executed and analysed as described before. Goal of the second study is answering research question 4: In what ways can teaching the concept of a compound variable quantity in different domains improve the understanding of the concept of a variable compound quantity?

The respondents in study 2 are one fifth grade teacher and his or her class. Given time and opportunity another teacher and class will be involved in this study starting another macro cycle of design research.

An important part of this research is the substitution of dynamic representations for students' images of function and calculus related concepts. In the first and subsequent teaching experiments dynamic representations are build, used and evaluated. To answer the third research question – how can dynamic representations support the teaching of the concept of a compound variable quantity? – we will reflect on the use of dynamic representations while teaching and learning the concept of compound variable quantities.

Finally a retrospective analysis of the whole process will be performed to answer all four research questions and come to a synthesis on the main research problem.

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2.4 Time plan

2.4.1 Detailed description of the work plan for the first twelve months

Activities	Period
Literature research	February 2010 – August 2010
Writing research proposal	April 2010 – January 2011
TU/e PROOF course on intercultural communication & cooperation	October 2010
ICO introductory course	September 2010 – November 2010
TU/e PROOF course on planning and communication	December 2010
ESoE Winterschool(Munich), 7-9 December 2010	November 2010 – December 2010
Developing experiment for preliminary study	August 2010 – February 2011
Writing educational software for use in experiment of preliminary study	August 2010 – February 2011

2.4.2 Global description of the work plan for the remaining duration of the project

Year Two

Activities	Period
First try-out of experiment of preliminary study	March 2011
Execution of one-on-one teaching experiments of the preliminary study	March 2011 – April 2011
Transcription of video recordings of the one-on-one teaching experiments	April 2011
Analysis of the experiments of the preliminary study	April 2011 – July 2011
Preparing poster for ORD	May 2011 – June 2011
Writing a scientific article on the results of the preliminary study	June 2011 – August 2011
Design and development of experiments of Study 1 based on the results of and reflection on the preliminary study	2011
Execution of experiments of Study 1: videotaping classroom experiments, interviewing pupils and teachers, transcribing video records, and collecting other data.	2011/2012

Year Three

Activities

Analysis of data gathered during experiments of Study 1

Writing article on findings of Study 1

Presenting paper on an international conference

Design and development of experiments of Study 2 based on the results of and reflection on study 1

Execution of experiments of Study 2: videotaping classroom experiments, interviewing pupils and teachers, transcribing video records, and collecting other data.

Analysis of data gathered during experiments of Study 2

Writing theoretical review article

Year Four

Activities

Writing article on findings of Study 2

Presenting paper on an international conference

Writing thesis

2.5 Setting within research group

In 1998 the Dutch government introduced the project VTB-Pro to improve science and technology education in primary education. As part of this project regional knowledge centers on science and technology education in primary education were set up. The ESoE is active partner in one of these regional knowledge centers: the Kenniscentrum Wetenschap en Techniek Zuid (KWTZ). This project is funded by the KWTZ and it fits the goals of the KWTZ to develop innovative science and technology education. The results of the project can help introducing the teaching of the concept of compound variable quantities and other calculus related concepts in primary education. It can form the basis for a re-schooling trajectory for primary teachers and teacher-students.

2.6 Output

- Presentation of work at different national (2) and international conferences (2)
- Five scientific articles
- PhD thesis based on these five articles

2.7 Societal and scientific relevance

The scientific relevance follows from the lack of research on teaching and learning about compound variable quantities in primary education. We do believe, however, that it is possible to do meaningful scientific research on this topic. With this research project we contribute to the field of mathematics education research in primary education.

Available research on teaching and learning concepts related to MCV in secondary education and up focusses on the domain of motion in particular. Teaching and learning of the concept of compound variable quantities in other domains is rare; research on transfer of understanding of compound variable quantities between different domains is even more rare.

This research will result in a theory on how to teach the concept of compound variable quantities in primary education. This theory is based on the idea to substitute dynamic representations for an image of calculus related concepts to enable learning and teaching of compound variable quantities in primary education. This method of substituting more abstract concepts by dynamic representations may be more general applicable to introduce other mathematical and scientific concepts new to primary education. Research on teaching and learning of such concepts in secondary education and up might be similarly adapted for use in developing innovative learning trajectories in primary education.

This study might also give an incentive to further innovate and integrate science and technology education in primary education. It could set an example for related projects.

Finally it has practical applications: the course materials developed can be used in Dutch primary education. Accompanying teacher training materials will also be developed.